

Black holes on the brane

Naresh Dadhich^{1,a}, Roy Maartens^{2,b}, Philippos Papadopoulos^{2,c} and Vahid Rezanian^{2,3,d}

¹*Inter-University Centre for Astronomy and Astrophysics, Ganeshkind, Pune 411 007, India*

²*Relativity and Cosmology Group, School of Computer Science and Mathematics, Portsmouth University, Portsmouth PO1 2EG, Britain*

³*Institute for Advanced Studies in Basic Sciences, Zanjan 45195, Iran*

We consider exact solutions for static black holes localized on a three-brane in five-dimensional gravity in the Randall-Sundrum scenario. We show that the Reissner-Nördstrom metric is an exact solution of the effective Einstein equations on the brane, re-interpreted as a black hole *without electric charge*, but with instead a *tidal ‘charge’* arising via gravitational effects from the fifth dimension. The tidal correction to the Schwarzschild potential is negative, which is impossible in general relativity, and in this case only *one* horizon is admitted, located outside the Schwarzschild horizon. The solution satisfies a closed system of equations on the brane, and describes the strong-gravity regime. Current observations do not strongly constrain the tidal charge, and significant tidal corrections could in principle arise in the strong-gravity regime and for primordial black holes.

04.50.+h, 04.70.-s

Phys. Lett. B, to appear

I. INTRODUCTION

Recent developments in string theory have shown that if matter fields are localized on a 3-brane in $1 + 3 + d$ dimensions, while gravity can propagate in the d extra dimensions, then the extra dimensions can be large (see, e.g., [1]). The extra dimensions need not even be compact, as in the 5-dimensional warped space models of Randall and Sundrum [2]. (See also [3] for earlier work.) In particular, they showed that it is possible to localize gravity on a 3-brane when there is one infinite extra dimension.

If matter on a 3-brane collapses under gravity, without rotating, to form a black hole, then the metric on the brane-world should be close to the Schwarzschild metric at astrophysical scales in order to preserve the observationally tested predictions of general relativity. Collapse to a black hole in the Randall-Sundrum brane-world scenario was studied by Chamblin et al. [4] (see also [5–8]). They gave a ‘black string’ solution which intersects the brane in a Schwarzschild solution.

Here we give an exact localized black hole solution, which remarkably has the mathematical form of the Reissner-Nördstrom solution, but *without electric charge* being present. Instead the Reissner-Nördstrom-type correction to the Schwarzschild potential can be thought of as a ‘tidal charge’, arising from the projection onto the brane of free gravitational field effects in the bulk. These effects are transmitted via the bulk Weyl tensor (see below). The Schwarzschild potential $\Phi = -M/(M_p^2 r)$, where M_p is the effective Planck mass on the brane, is modified to

$$\Phi = -\frac{M}{M_p^2 r} + \frac{Q}{2r^2}, \quad (1)$$

where the constant Q is a ‘tidal charge’ parameter, which may be positive or negative.

A geometric approach to the Randall-Sundrum scenario has been developed by Shiromizu et al. [9] (see also [10]), and proves to be a useful starting point for formulating the problem and seeing clear lines of approach. The field equations in the bulk are (modifying the notation of [9])

$$\tilde{G}_{AB} = \tilde{\kappa}^2 \left[-\tilde{\Lambda} \tilde{g}_{AB} + \delta(\chi) (-\lambda g_{AB} + T_{AB}) \right], \quad (2)$$

where the tildes denote bulk quantities. The fundamental 5-dimensional Planck mass \tilde{M}_p enters via $\tilde{\kappa}^2 = 8\pi/\tilde{M}_p^3$. The brane tension is λ , and $\tilde{\Lambda}$ is the bulk cosmological constant. The brane is located at $\chi = 0$ (so that $x^4 = \chi$ is a natural choice for the fifth dimension coordinate), and $g_{AB} = \tilde{g}_{AB} - n_A n_B$ is the induced metric on the brane, with n_A the spacelike unit normal to the brane. The brane energy-momentum tensor is T_{AB} , and $T_{AB} n^B = 0$. The brane is a fixed point of the Z_2 symmetry.

II. FIELD EQUATIONS ON THE BRANE

The field equations induced on the brane arise from Eq. (2), the Gauss-Codazzi equations and the matching conditions with Z_2 -symmetry, and they may be written as a modification of the standard Einstein equations, with the new terms carrying bulk effects onto the brane [9]:

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + \kappa^2 T_{\mu\nu} + \tilde{\kappa}^4 S_{\mu\nu} - \mathcal{E}_{\mu\nu}, \quad (3)$$

where $\kappa^2 = 8\pi/M_p^2$. The energy scales are related to each other and to the cosmological constants via

$$M_p = \sqrt{\frac{3}{4\pi}} \left(\frac{\widetilde{M}_p^2}{\sqrt{\lambda}} \right) \widetilde{M}_p, \quad \Lambda = \frac{4\pi}{\widetilde{M}_p^3} \left[\widetilde{\Lambda} + \left(\frac{4\pi}{3\widetilde{M}_p^3} \right) \lambda^2 \right]. \quad (4)$$

Typically, the fundamental Planck scale is much lower than the effective scale in the brane-world: $\widetilde{M}_p \ll M_p$. Local bulk effects on the matter are transmitted via the ‘squared energy-momentum’ tensor $S_{\mu\nu}$, but since we will consider vacuum solutions, the precise form of $S_{\mu\nu}$ (see [9]) will not be needed. In vacuum, $T_{\mu\nu} = 0 = S_{\mu\nu}$, and we also choose the bulk cosmological constant to satisfy $\widetilde{\Lambda} = -4\pi\lambda^2/3\widetilde{M}_p^3$, so that $\Lambda = 0$ by Eq. (4). Then Eq. (3) reduces to

$$R_{\mu\nu} = -\mathcal{E}_{\mu\nu}, \quad R_\mu{}^\mu = 0 = \mathcal{E}_\mu{}^\mu, \quad (5)$$

where $\mathcal{E}_{\mu\nu}$ is the limit on the brane of the projected bulk Weyl tensor [9]:

$$\mathcal{E}_{AB} = \widetilde{C}_{ACBD} n^C n^D. \quad (6)$$

The Weyl symmetries ensure that this is symmetric and tracefree ($\mathcal{E}_{[AB]} = 0 = \mathcal{E}_A{}^A$) and has no orthogonal components ($\mathcal{E}_{AB} n^B = 0$, so that $\mathcal{E}_{AB} \rightarrow \mathcal{E}_{\mu\nu} \delta_A{}^\mu \delta_B{}^\nu$ as $\chi \rightarrow 0$). It carries the influence of nonlocal gravitational degrees of freedom in the bulk onto the brane, including the tidal (or Coulomb) and transverse traceless (gravitational wave) aspects of the free gravitational field. (See [11] for a fuller discussion of $\mathcal{E}_{\mu\nu}$ in the general case.) On the brane, in the vacuum case, this tensor satisfies the divergence constraint [9]

$$\nabla^\mu \mathcal{E}_{\mu\nu} = 0, \quad (7)$$

where ∇_μ is the brane covariant derivative. In view of the Bianchi identities on the brane, this is an integrability condition for the field equation $R_{\mu\nu} = -\mathcal{E}_{\mu\nu}$. For static solutions, Eqs. (5) and (7) form a *closed system of equations on the brane* [11].

A vacuum solution outside a mass localized on the brane must satisfy equations (5) and (7). This leads to a prescription for mapping 4-dimensional general relativity solutions to brane-world solutions in 5-dimensional gravity: *a stationary general relativity solution with trace-free energy-momentum tensor gives rise to a vacuum brane-world solution in 5-dimensional gravity*. The 4-dimensional general relativity energy-momentum tensor $T_{\mu\nu}$ (where $T_\mu{}^\mu = 0$) is formally identified with the bulk Weyl term on the brane via the correspondence

$$\kappa^2 T_{\mu\nu} \longleftrightarrow -\mathcal{E}_{\mu\nu}.$$

The general relativity conservation equations $\nabla^\nu T_{\mu\nu} = 0$ correspond to the constraint equation (7) on the brane. In particular, Einstein-Maxwell solutions in general relativity will lead to vacuum brane-world solutions. This is the observation that led us to the Reissner-Nördstrom-type solution.

III. SOLUTIONS WITH TIDAL CHARGE

Algebraic symmetry properties imply that in general we can decompose $\mathcal{E}_{\mu\nu}$ irreducibly with respect to a chosen 4-velocity field u^μ as [11]

$$\mathcal{E}_{\mu\nu} = -\left(\frac{\kappa}{\widetilde{\kappa}}\right)^4 [\mathcal{U} (u_\mu u_\nu + \frac{1}{3} h_{\mu\nu}) + \mathcal{P}_{\mu\nu} + 2\mathcal{Q}_{(\mu} u_{\nu)}], \quad (8)$$

where $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$ projects orthogonal to u^μ . Here

$$\mathcal{U} = -\left(\frac{\kappa}{\widetilde{\kappa}}\right)^4 \mathcal{E}_{\mu\nu} u^\mu u^\nu$$

is an effective energy density on the brane arising from the free gravitational field in the bulk—but note that this energy density need not be positive. Indeed, as we argue below, $\mathcal{U} < 0$ is the natural case. The effective anisotropic stress from the free gravitational field in the bulk is the spatially tracefree and symmetric part, i.e.,

$$\mathcal{P}_{\mu\nu} = -\left(\frac{\kappa}{\widetilde{\kappa}}\right)^4 [h_{(\mu}{}^\alpha h_{\nu)}{}^\beta - \frac{1}{3} h_{\mu\nu} h^{\alpha\beta}] \mathcal{E}_{\alpha\beta},$$

where round brackets denote symmetrization. The effective energy flux from the free gravitational field in the bulk is

$$\mathcal{Q}_\mu = \left(\frac{\kappa}{\widetilde{\kappa}}\right)^4 h_\mu{}^\alpha \mathcal{E}_{\alpha\beta} u^\beta.$$

In a static vacuum, with u^μ along the static Killing direction, we have $\mathcal{Q}_\mu = 0$, and the effective conservation equation (7) reduces to the single spatial equation

$$\frac{1}{3} D_\mu \mathcal{U} + \frac{4}{3} \mathcal{U} A_\mu + D^\nu \mathcal{P}_{\mu\nu} + A^\nu \mathcal{P}_{\mu\nu} = 0, \quad (9)$$

where D_μ is the projection (orthogonal to u^μ) of the brane covariant derivative [12], and $A_\mu = u^\nu \nabla_\nu u_\mu$ is the 4-acceleration. Static spherical symmetry means that

$$A_\mu = A(r) r_\mu, \quad \mathcal{P}_{\mu\nu} = \mathcal{P}(r) [r_\mu r_\nu - \frac{1}{3} h_{\mu\nu}],$$

for some functions $A(r)$ and $\mathcal{P}(r)$, where r is the areal distance and r_μ is a unit radial vector. The Reissner-Nördstrom-type solution in Eq. (1) corresponds to the solution

$$\mathcal{U} = \left(\frac{\kappa}{\widetilde{\kappa}}\right)^4 \frac{Q}{r^4} = -\frac{1}{2} \mathcal{P} \quad (10)$$

of Eq. (9).

We can verify that the solution in Eqs. (1) and (10) satisfies the field equations (5), using natural coordinates, for which the metric on the brane is

$$ds^2 = -A(r) dt^2 + B(r) dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2). \quad (11)$$

Then it may be verified that

$$A = B^{-1} = 1 + \frac{\alpha}{r} + \frac{\beta}{r^2}, \quad (12)$$

$$\mathcal{E}_t^t = \mathcal{E}_r^r = -\mathcal{E}_\theta^\theta = -\mathcal{E}_\varphi^\varphi = \frac{\beta}{r^4}, \quad (13)$$

where α and β are constants. Equations (12) and (13) satisfy all the field equations (5), and hence also the divergence equations (7). By Eqs. (8) and (10), we see that $\beta = Q$, and the far-field Newtonian limit [see Eq. (1)] shows that $\alpha = -2M/M_p^2$.

In summary, we have shown that *an exact black hole solution of the effective field equations on the brane is given by the induced metric*

$$-g_{tt} = (g_{rr})^{-1} = 1 - \left(\frac{2M}{M_p^2} \right) \frac{1}{r} + \left(\frac{q}{M_p^2} \right) \frac{1}{r^2}, \quad (14)$$

where $q = Q\widetilde{M}_p^2$ is a dimensionless tidal charge parameter. The projected Weyl tensor, transmitting the tidal charge stresses from the bulk to the brane, is

$$\mathcal{E}_{\mu\nu} = - \left(\frac{q}{M_p^2} \right) \frac{1}{r^4} [u_\mu u_\nu - 2r_\mu r_\nu + h_{\mu\nu}]. \quad (15)$$

IV. PROPERTIES OF THE BLACK HOLE

The 4-dimensional horizon structure of the brane-world black hole depends on the sign of q . For $q \geq 0$, there is a direct analogy to the Reissner-Nördstrom solution, with two horizons:

$$r_\pm = \frac{M}{M_p^2} \left[1 \pm \sqrt{1 - q \frac{M_p^4}{M^2 M_p^2}} \right]. \quad (16)$$

As in general relativity, both horizons lie inside the Schwarzschild horizon: $0 \leq r_- \leq r_+ \leq r_s = 2M/M_p^2$, and there is an upper limit on q :

$$0 \leq q \leq q_{\max} = \left(\frac{\widetilde{M}_p}{M_p} \right) \left(\frac{M}{M_p} \right)^2. \quad (17)$$

The intriguing new possibility that $q < 0$, which is impossible in the general relativity Reissner-Nördstrom case, leads to only *one horizon, lying outside the Schwarzschild horizon*:

$$r_+ = \frac{M}{M_p^2} \left[1 + \sqrt{1 - q \frac{M_p^4}{M^2 M_p^2}} \right] > r_s. \quad (18)$$

In the $q < 0$ case, the (single) horizon has a greater area than its Schwarzschild counterpart, so that bulk effects act to *increase the entropy and decrease the temperature of the black hole*. In general relativity, the electric field in the Reissner-Nördstrom solutions acts to *weaken*

the gravitational field, and the same is true for the brane-world black hole with $q > 0$. This can be seen clearly from Eq. (1). By contrast, the $q < 0$ case corresponds to the opposite effect, i.e., bulk effects tend to *strengthen the gravitational field*. By Eq. (10), we see that in this case, the effective energy density \mathcal{U} on the brane contributed by the free gravitational field in the bulk is *negative*. This is in accord with the (Newtonian) notion that the gravitational field of an isolated mass has negative energy density. Furthermore, it agrees with the perturbative analysis by Sasaki et al. [13] and the non-perturbative analysis of Maartens [11]. The gravitational field generated by a source on the brane tends to squeeze test matter in the 5th direction, thus acting as an attractive field with a negative energy contribution. The tidal acceleration measured by static observers along the 5th direction n^A is [11]

$$-\widetilde{R}_{ABCD}u^A n^B u^C n^D = \left(\frac{\widetilde{\kappa}}{\kappa} \right)^4 \mathcal{U} + \frac{1}{6} \widetilde{\kappa}^2 \widetilde{\Lambda},$$

where the right hand side follows from Eqs. (2) and (6). The negative bulk cosmological constant contributes to acceleration towards the brane, reflecting its confining role on the gravitational field. In order for \mathcal{U} to reinforce confinement, it must be negative. In other words, negative tidal charge $q < 0$ is the physically more natural case. (See also [14] for further discussion of negative energy density from bulk effects.) Furthermore, $q < 0$ ensures that the singularity is spacelike, as in the Schwarzschild solution, whereas $q > 0$ leads to a timelike singularity, which amounts to a qualitative change in the nature of the general relativistic Schwarzschild solution.

It is widely assumed that astrophysical black holes cannot exhibit macroscopic electric charge due to the presence of neutralizing plasma in their vicinity. Discussions of astrophysical black hole phenomena are therefore commonly restricted to the Kerr family of black holes. The solution presented here raises the possibility that an effective Reissner-Nördstrom metric emerges by geometric considerations and is hence not constrained by the above argument. The tidal charge q affects the geodesics and the gravitational potential, so that indirect limits may be placed on it by observations. Current observational limits on $|q|$ are rather weak, since the correction term in Eq. (1) dies off rapidly with increasing r , and astrophysical measurements (lensing and perihelion precession) probe mostly (weak-field) solar scales. These measurements require the correction term in the gravitational potential to be much less than the Schwarzschild term, so that

$$|q| \ll 2 \left(\frac{\widetilde{M}_p}{M_p} \right)^2 M_\odot R_\odot. \quad (19)$$

This still allows for large values of $|q|$, which would modify the spacetime geometry of a nonrotating black hole in

the strong-gravity regime [15], with implications for example for the last stable circular orbit for compact binaries. Although the strong-gravity regime is not currently directly accessible to observations, indirect limits could emerge from the way in which tidal charge modifies general relativity strong-gravity effects. This deserves further investigation.

Further indirect limits could arise from the effect of tidal charge on primordial black holes, which also merits further analysis. An intriguing possibility is that strong tidal effects in the early universe could lead to black hole formation even in the absence of gravitational collapse of matter. If such matter-free tidal collapse is possible, then the endstate is a metric with $M = 0$ and $q < 0$:

$$-g_{tt} = (g_{rr})^{-1} = 1 + \left(\frac{q}{M_p^2} \right) \frac{1}{r^2}, \quad (20)$$

and the horizon on the brane is given by

$$r_h = \frac{\sqrt{-q}}{\widetilde{M}_p}. \quad (21)$$

The tidal charge parameter arises from purely scalar (Coulomb-like) effects of the free gravitational field in the bulk, given the static spherical symmetry. In the original Randall-Sundrum model, as well as in perturbative studies of it (see, e.g., [2,6,8,13]), the bulk is assumed to be exactly anti-de Sitter in the absence of any source on the brane. The geometric approach of [9], which we have adopted here, makes no assumptions about the bulk metric, other than that it satisfies the 5-dimensional Einstein equations with cosmological constant. Thus the bulk need not be conformally flat in the absence of sources on the brane. This means that in general, *the tidal charge parameter q will be determined by both the brane source, i.e., the mass M , and any Coulomb part of the bulk Weyl tensor that survives when M is set to zero.*

V. CONCLUSION

We have not investigated fully the effect of the brane-world black hole on the bulk geometry, and in particular the nature of the off-brane horizon structure. This has been done for solutions which reduce to the Schwarzschild black hole on the brane [4]. In these solutions, the bulk metric is

$$d\tilde{s}^2 = - \left(\frac{6}{\tilde{\kappa}^2 \tilde{\Lambda}} \right) \frac{1}{z^2} [dz^2 + g_{\mu\nu}(x^\alpha) dx^\mu dx^\nu],$$

where $g_{\mu\nu}$ is the Schwarzschild metric. We have adopted a different approach: instead of starting from an induced Schwarzschild metric, we have solved the effective field equations for the induced metric on the brane (which form a closed system, since the metric is static), and found a generalization of the Schwarzschild solution.

It turns out that if ds^2 is given by our solution, as in Eqs. (11)–(14), then

$$d\tilde{s}^2 = f(z) [dz^2 + ds^2]$$

cannot satisfy the bulk field equations for any $f(z)$ if $q \neq 0$. Finding an exact form of the bulk metric that is consistent with our exact induced metric on the brane is more difficult than the case where the induced metric is Schwarzschild.

Perturbative studies, which start from an exactly anti-de Sitter background, show that the first weak-field correction of the Newtonian potential on the brane is proportional to $1/r^3$ (see [2,6,8,13]). Our solution has by contrast a $1/r^2$ correction, so that it is incompatible with the long-distance limit on the brane. (Such a correction can also arise in thick-brane models [8].) However, in the *short-distance* limit on the brane, the lowest order correction to the potential is proportional to $1/r^2$ [16]. This term will dominate the $1/r$ term, which reflects the fact that gravity becomes effectively 5-dimensional at high energies. Thus our solution should describe well the strong-gravity regime on the brane. In the short-distance limit, the perturbative analysis shows that [16]

$$q = - \frac{M}{\widetilde{M}_p},$$

so that the tidal charge is negative, as we argued above, and is determined by the black hole mass (which is to be expected if the background bulk Weyl tensor vanishes).

In order to pursue our nonperturbative analysis, we need to look at the off-brane equations for the curvature (see [13] for the general form of these equations). The induced field equations on the brane given in Eq. (5) are supplemented by off-brane equations which determine

$$\mathcal{L}_n \mathcal{E}_{AB}, \mathcal{L}_n \mathcal{B}_{ABC}, \mathcal{L}_n R_{ABCD},$$

where \mathcal{L}_n is the Lie derivative along n^A , R_{ABCD} is the 4-dimensional Riemann tensor, and

$$\mathcal{B}_{ABC} = g_A^D g_B^E \tilde{C}_{DEC} n^F,$$

with $\mathcal{B}_{\mu\nu\sigma} = 0$ on the brane. These equations together with Eq. (5) form a closed system [13].

In conclusion, we have shown how a Reissner-Nördstrom-type metric satisfies the effective field equations on the brane in Randall-Sundrum-type gravity, which form a closed system because of staticity. There is no electric charge present, but instead a tidal charge, arising from the imprint of the free gravitational field in the bulk on the brane. The tidal charge correction to the Schwarzschild potential is negative, and the solution describes the strong-gravity regime on the brane. Negative q leads to an horizon on the brane that is outside the Schwarzschild horizon, corresponding to lower temperature and greater entropy. Current observations place only weak limits on the tidal charge, and in principle significant tidal modifications could arise in the strong-field

regime or in the early-universe case of primordial black holes. Further investigation is in progress to probe these modifications and any indirect limits that they may impose on the tidal charge, as well as to find the off-brane behaviour of the horizon.

Acknowledgements: VR was supported by a Royal Society grant while at Portsmouth, and thanks the Relativity and Cosmology Group for hospitality. RM thanks IUCAA, Pune for hospitality during a visit, which was partially supported by the Royal Society. We thank Bruce Bassett, Marco Bruni, Roberto Casadio, Roberto Emparan, David Langlois, Jose Senovilla, Carlo Ungarelli and David Wands for useful discussions and comments, and especially Tetsuya Shiromizu.

- [8] C. Csaki, J. Erlich, T.J. Hollowood, and Y. Shirman, Nucl. Phys. **B581**, 309 (2000).
- [9] T. Shiromizu, K. Maeda, and M. Sasaki, Phys. Rev. D **62**, 024012 (2000).
- [10] P. Binetruy, C. Deffayet, U. Ellwanger, and D. Langlois, Phys. Lett. **B477**, 285 (2000).
- [11] R. Maartens, Phys. Rev. D, to appear (2000) (hep-th/0004166).
- [12] R. Maartens, Phys. Rev. D **55**, 463 (1997);
R. Maartens and B.A. Bassett, Class. Quantum Grav. **15**, 705 (1998).
- [13] M. Sasaki, T. Shiromizu, and K. Maeda, Phys. Rev. D **62**, 024008 (2000).
- [14] D. Vollick, hep-th/0004064.
- [15] S. Chandrasekhar, *The Mathematical Theory of Black Holes* (Oxford University Press: 1983).
- [16] T. Shiromizu, private communication.

^a nkd@iucaa.ernet.in

^b roy.maartens@port.ac.uk

^c philippos.papadopoulos@port.ac.uk

^d rezaia@iasbs.ac.ir

- [1] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998);
I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **436**, 257 (1998).
- [2] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 and 4690 (1999).
- [3] K. Akama, hep-th/0001113 (reproduced from 1982 article);
V.A. Rubakov and M.E. Shaposhnikov, Phys. Lett. **125B**, 139 (1983);
M. Visser, Phys. Lett. **B159**, 22 (1985);
M. Gogberashvili, Europhys. Lett. **49**, 396 (2000).
- [4] A. Chamblin, S.W. Hawking, and H.S. Reall, Phys. Rev. D **61**, 065007 (2000).
- [5] P.C. Argyres, S. Dimopoulos, and J. March-Russell, Phys. Lett. **B441**, 96 (1998);
P. Kraus, JHEP **9912**, 011 (1999);
J. Garriga and M. Sasaki, hep-th/9912118;
A. Chamblin, C. Csaki, J. Erlich, and T.J. Hollowood, Phys. Rev. D **62**, 044012 (2000);
R. Emparan, G.T. Horowitz, and R.C. Myers, hep-th/0003118;
S. Nojiri, O. Obregon, S.D. Odintsov, and S. Ogushi, hep-th/0003148;
R. Casadio and B. Harms, Phys. Lett. B, to appear (2000) (hep-th/0004004).
- [6] S.B. Giddings, E. Katz, and L. Randall, JHEP **0003**, 023 (2000);
J. Garriga and T. Tanaka, Phys. Rev. Lett. **84**, 2778 (2000).
- [7] R. Emparan, G.T. Horowitz, and R.C. Myers, JHEP **0001**, 007 (2000).